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$\mathcal{N} = 1$ matter from fractional branes*

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Abstract: We study a bound state of fractional D3-branes localized inside the world-volume of fractional D7-branes on the orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. We determine the open string spectrum that leads to $\mathcal{N} = 1$ $\mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \mathrm{U}(N_3) \times \mathrm{U}(N_4)$ gauge theory with matter having the number of D7-branes as a flavor index. We derive the linearized boundary action of the D7-brane on this orbifold using the boundary state formalism and we discuss the tadpole cancellation. After computing the asymptotic expression of the supergravity solution the anomalies of the gauge theory are reproduced.

Keywords: D-branes, String Duality, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

An efficient way to study the relation between String Theory and Standard Model (SM) field theory is through the *bottom-up* approach [1] which constitutes an alternative to the traditional *top-down* approach. In the latter one starts from a string theory and tries to reproduce the SM massless spectrum by a suitable Calabi-Yau manifold compactification that reduces the number of space-time dimensions and supersymmetry, giving the right gauge group. In the former, instead, one looks for configurations of D-branes with world-volume gauge theories similar as much as possible to the SM. One of the advantages of this approach is that it does not require any knowledge of the geometrical details of the manifold compactification. We would like here to observe that the bottom-up approach is essentially based on the property that a D-brane supports a gauge field theory on its world-volume. On the other hand it is well-known that D-branes can be studied from a complementary point of view: they are classical solutions of the ten-dimensional supergravity. This is the essence of the *gauge/gravity* correspondence. It means that one can analyze the classical backgrounds generated by a particular configuration of D-branes to get some insight on the *dual* gauge theories living on their world-volume and viceversa. This is just the point of view we want to adopt in this paper that can represent a sort of link between the bottom-up approach and the gauge/gravity correspondence. We study the latter by considering a suitable configuration of fractional branes yielding the SM-like gauge theory. Fractional branes,[1] living at orbifold singularities, are interesting tools to study gauge/gravity correspondence for non conformal gauge theories with reduced supersymmetry [2, 3] in different renormalization schemes [4]. This aim would be better achieved, for

[1]For a recent review on fractional branes see ref. [2] and references therein.

$\mathcal{N} = 1$ theories, by considering branes wrapped on supersymmetric cycles inside a Calabi-Yau space. In fact it has been shown that the classical supergravity solutions determined by these non perturbative objects are completely regular [5, 6] and therefore they allow to study also the infrared properties of the dual gauge theories [7, 8, 9, 10]. Unfortunately, within this approach, it is not straightforward to find a classical background dual to gauge theory with chiral matter, which is instead more easily deducible from bound states of fractional branes. We expect, in analogy with similar configurations, that the classical background determined by fractional branes [11]–[16] contains a naked singularity resolved by the *enhançon* mechanism [17]. The enhançon is a sort of screen which excises the region of space-time corresponding to scales where non-perturbative effects become relevant in the gauge theory and this clarifies why we are able to obtain only the ultraviolet behavior of the field theory.

In this paper we consider a stack of fractional D3-D7 branes living on the orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. The choice of such an orbifold is motivated by the interest in $\mathcal{N} = 1$ supersymmetric gauge theories with matter. In fact this is one of the simplest orbifolds preserving four supersymmetry charges. Furthermore in this orbifold background there are four different kinds of fractional D-branes corresponding to the four irreducible representations of the abelian discrete group $\mathbb{Z}_2 \times \mathbb{Z}_2$ [18, 19]. The gauge theory supported by a general bound state of N_I fractional branes of type I is a $\bigotimes_{I=1}^{4} \mathrm{U}(N_I)$ gauge theory with chiral multiplets charged under the bifundamental representations of the gauge group. The inclusion of fractional D7-branes, since the open strings have one end on a D3-brane and the other end on a D7, allows us to have chiral multiplets charged only under the fundamental representation of the gauge group. They have also a flavor index running on the number of fractional D7-branes.

It is relevant to note that when we turn off three of the gauge groups by choosing $N_2 = N_3 = N_4 = 0$ the gauge theory (in the presence of the D7-brane) is exactly $\mathcal{N} = 1$ super QCD with matter.

In the forthcoming discussion we will consider a bound state of a D3-brane living at the orbifold singularities and hence it is completely localized in the world-volume of the D7-brane. The latter is partially extended along the orbifold directions. For our purposes it has been enough to compute the asymptotic behavior of the classical supergravity solution via the boundary state formalism [20, 21].

The solution so found exhibits a U(1) isometry acting as a phase factor on the coordinates transverse to the D3 branes. Such an invariance is related to the classical $\hat{\mathrm{U}}(1)_R$ R-symmetry of the dual gauge theory [22]. In a gauge theory this symmetry is just a member of a supersymmetric multiplet containing also the dilations. In classical theories, which are invariant under superconformal transformations, the corresponding currents are conserved. Quantum corrections break superconformal invariance and give rise to anomalies. The anomaly breaking of the global $\hat{\mathrm{U}}(1)_R$ symmetry appears as a spontaneous symmetry breaking in supergravity [23]. It is important to stress that the $\hat{\mathrm{U}}(1)_R$ is not the final anomaly free $\mathrm{U}(1)_R$ which is obtained via a suitable choice of the R-charges of the various fields. We have found that also our classical solution explicitly breaks the U(1) isometry to one of its discrete subgroups. Therefore we expect to obtain the associated anomaly of the underlying gauge theory. In fact, by analyzing on the supergravity side how the solution transforms under a combined scale and chiral transformation, we have been able to reproduce the corresponding anomalies, i.e. the one-loop β-function and its chiral anomaly. It is evident that we have used supergravity to get perturbative information on the dual gauge theory. This means that our correspondence has to be interpreted not in the same spirit as AdS/CFT, but more properly in that of open/closed string duality.

The paper is organized as follows.

In section 2 we analyze the properties of fractional D3-branes on the orbifold $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$ and in particular we study how the orbifold acts on the open strings stretched between two of these branes. This analysis allows us to obtain the massless spectrum of the gauge theory surviving the orbifold projection.

Furthermore we introduce fractional D7-branes, determine the massless spectrum of the matter chiral fields and fix a configuration of D7-branes that leads to an anomaly free gauge theory.

In section 3 we analyze, from the point of view of the boundary state approach, the boundary action of our system. In particular, by writing the boundary states of a fractional D7-brane and by computing the coupling of this boundary with the bulk fields, we construct, at linearized level, the boundary action for such a brane in this orbifold. Furthermore, still within the boundary state formalism, we also compute the classical solution of the equations of motion at first order in the string coupling g_s. In the last part of this section we investigate the stability of the system related to the one-loop massless tadpole cancellation.

Finally in section 4, by studying how the scale and chiral transformations of the dual gauge theory are realized in supergravity, we are able to compute the corresponding anomalies. The one-loop β-function and the chiral anomalies emerge out from the asymptotic classical profile of the bulk fields.

2. D3 and D7-branes on the orbifold $\mathbb{R}^{1,3}\times\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$

Let us consider a system of D3 and D7-branes of type-IIB string theory on the orbifold $\mathbb{R}^{1,3}\times\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$. In particular we analyze the spectrum of the massless open string states having their end-points attached to them. We take the orbifold directions to be along $x^4,\ldots,x^9$.

2.1 General features of the orbifold $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$

The six coordinates of the orbifold space can be simply arranged in three complex coordinates spanning $\mathbb{C}^3$:

$$z^1 = x^4 + ix^5\,,\qquad z^2 = x^6 + ix^7\,,\qquad z^3 = x^8 + ix^9\,. \tag{2.1}$$

The $\mathbb{Z}_2$ group is characterized by two elements $\{\mathbf{1},h\}$, with $h^2 = \mathbf{1}$, hence the tensor product $\mathbb{Z}_2\times\mathbb{Z}_2$ is made of the following four elements

$$\mathbb{Z}_2\times\mathbb{Z}_2\,:\,\{e=\mathbf{1}\times\mathbf{1}\,,\ h_1 = h\times\mathbf{1}\,,\ h_2=\mathbf{1}\times h\,,\ h_3 = h\times h\}\,. \tag{2.2}$$

The action of the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ on the complex vector $\vec{z} = (z^1, z^2, z^3)$ of $\mathbb{C}^3$ is given by a three-dimensional representation (necessarily reducible) of this finite group. We perform the following choice:

$$R_e = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad R_{h_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$
$$R_{h_2} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad R_{h_3} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2.3}$$

It is possible to summarize the orbifold action on $\mathbb{C}^3$ as follows:

$$R_{h_i} = \mathrm{diag}\left[e^{i\pi b^1_{h_i}}, e^{i\pi b^2_{h_i}}, e^{i\pi b^3_{h_i}}\right], \tag{2.4}$$

with

$$\vec{b}_{h_1} = (0,1,1)\,, \qquad \vec{b}_{h_2} = (1,0,1)\,, \qquad \vec{b}_{h_3} = (1,1,0)\,. \tag{2.5}$$

The vectors defined in (2.5) clearly satisfy the following condition:

$$b^1_{h_i} + b^2_{h_i} + b^3_{h_i} = 0 \bmod 2\,. \tag{2.6}$$

The present choice of the orbifold representation from the point of view of the gauge theory is the one which leaves intact the $\mathcal{N} = 1$ theory we are interested in.

The spectrum of the closed string theory on the orbifold space $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ consists of an untwisted sector, corresponding to the identity of the orbifold group and three twisted sectors corresponding to its non trivial elements. In particular, the massless spectrum in the twisted sectors coincides with the zero modes of supergravity fields dimensionally reduced on the three exceptional vanishing two-cycles $\mathcal{C}_i$ with $i = 1, 2, 3$ characterizing the orbifold, each of them embedded in one of the three four-dimensional subspaces of $\mathbb{C}^3$. The three anti self-dual two-forms ω^i_2, dual to the cycles $\mathcal{C}_i$, are completely independent and normalized as:[2]

$$\int_{\mathcal{C}_i} \omega^j_2 = 2\delta^i_j\,, \qquad \int \omega^i_2 \wedge \omega^i_2 = -1\,, \qquad *_4\omega^i_2 = -\omega^i_2\,, \tag{2.7}$$

where $*_4$ denotes the dual in the four-dimensional space in which the two-cycle is embedded.

In the following we are interested in describing fractional branes coupled to two kinds of twisted scalars and twisted RR four-form. The scalars are obtained by reducing the Kalb-Ramond two-form B_2 and the RR two-form on the exceptional two-cycles, while the RR four-form is obtained by reducing the RR six-form, i.e.:

$$B_2 = \sum_{i=1}^{3} b^i \omega^i_2\,; \qquad C_2 = \sum_{i=1}^{3} c^i \omega^i_2\,; \qquad C_6 = \sum_{i=1}^{3} A^i_4 \wedge \omega^i_2\,. \tag{2.8}$$

[2]In this paper we adopt a different convention with respect to the paper of ref. [19]. One can go from one convention to the other by rescaling by a factor 2 the antiself-dual two-forms: $\omega = 2\omega_{[19]}$.

2.2 D3-D3 open string spectrum on the orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

Let us introduce now fractional D3-branes on this orbifold. In particular we are interested in studying configurations where these branes are transverse to the orbifold, namely with world-volume directions x^μ with $\mu = 0, 1, 2, 3$.

Before the orbifold projection, the low energy modes consistent with the open superstring theory living on the N D3-branes, are those of the four dimensional $\mathcal{N} = 4$ U(N) super Yang Mills theory. In fact a generic open string state is the product of a Chan-Paton factor, that is in this case an $N \times N$ matrix, and of an oscillator part. In particular, in the Neveu-Schwarz sector, the massless states are given by:

$$\begin{aligned} A^\mu &= \lambda \otimes \psi^\mu_{-1/2} |0, k\rangle \,, \\ \phi^i &= \lambda \otimes \left(\psi^{2i+2}_{-1/2} + i \psi^{2i+3}_{-1/2} \right) |0, k\rangle \end{aligned} \tag{2.9}$$

where λ denotes the Chan-Paton factor and $i = 1, 2, 3$.

The states A^μ can be identified with the U(N) gauge bosons of the supersymmetric $\mathcal{N} = 4$ gauge theory; the three complex fields ϕ^i describe the six real scalars living, as A^μ, in the adjoint representation of the U(N) gauge group. All of them fill the bosonic part of the $\mathcal{N} = 4$ gauge multiplet.

The same analysis can be repeated for the Ramond sector. In this case the massless spectrum is identified with the four two-component (on shell) fermions still in the adjoint representation of U(N).

Let us now consider the spectrum in the orbifold theory, taking into account that the orbifold group acts both on the oscillators and on the Chan-Paton factors. The only allowed states are the ones surviving under this combined action. We would like here to remind that the action of the orbifold group element h on the Chan-Paton factors is defined as:

$$\gamma(h) \lambda \gamma^{-1}(h) = \lambda' \,. \tag{2.10}$$

In the case of $\mathbb{Z}_2 \times \mathbb{Z}_2$, h runs over the four elements of this group defined in (2.2).

We are interested in *fractional* branes which are defined as D-branes whose Chan-Paton factors transform under the irreducible representations of the orbifold group. $\mathbb{Z}_2 \times \mathbb{Z}_2$ has four one-dimensional irreducible representations:

$$\begin{array}{llll} \gamma_1(e) = +1 & \gamma_1(h_1) = +1 & \gamma_1(h_2) = +1 & \gamma_1(h_3) = +1 \,, \\ \gamma_2(e) = +1 & \gamma_2(h_1) = +1 & \gamma_2(h_2) = -1 & \gamma_2(h_3) = -1 \,, \\ \gamma_3(e) = +1 & \gamma_3(h_1) = -1 & \gamma_3(h_2) = +1 & \gamma_3(h_3) = -1 \,, \\ \gamma_4(e) = +1 & \gamma_4(h_1) = -1 & \gamma_4(h_2) = -1 & \gamma_4(h_3) = +1 \,. \end{array} \tag{2.11}$$

The above definition implies that fractional branes, differently from the regular ones, do not admit images and hence they live at the orbifold fixed point $z_1 = z_2 = z_3 = 0$. Furthermore it also shows that there are four different kinds of fractional branes that we label with the index $I = 1, 2, 3, 4$. In the following we consider a configuration made of N_I D3-branes of type I, with $\sum_{I=1}^{4} N_I = N$. The orbifold action in (2.11) is generalized to:

$$\gamma_e^{(D3)} = \mathrm{diag}(\mathbf{1}_{N_1,N_1}, \mathbf{1}_{N_2,N_2}, \mathbf{1}_{N_3,N_3}, \mathbf{1}_{N_4,N_4})\,, \tag{2.12}$$

$$\gamma_{h_1}^{(D3)} = \mathrm{diag}(\mathbf{1}_{N_1,N_1}, \mathbf{1}_{N_2,N_2}, -\mathbf{1}_{N_3,N_3}, -\mathbf{1}_{N_4,N_4})\,, \tag{2.13}$$

$$\gamma_{h_2}^{(D3)} = \mathrm{diag}(\mathbf{1}_{N_1,N_1}, -\mathbf{1}_{N_2,N_2}, \mathbf{1}_{N_3,N_3}, -\mathbf{1}_{N_4,N_4})\,, \tag{2.14}$$

$$\gamma_{h_3}^{(D3)} = \mathrm{diag}(\mathbf{1}_{N_1,N_1}, -\mathbf{1}_{N_2,N_2}, -\mathbf{1}_{N_3,N_3}, \mathbf{1}_{N_4,N_4})\,. \tag{2.15}$$

Our orbifold action preserves only the $\mathcal{N}=1$ supersymmetry while acting on the states of the NS-sector as follows:

$$\left[\gamma_{h_l}^{(D3)}\right] A^{\mu} \left[\gamma_{h_l}^{(D3)}\right]^{-1} = A^{\mu}\,, \tag{2.16}$$

$$\left[\gamma_{h_l}^{(D3)}\right] \phi^{i} \left[\gamma_{h_l}^{(D3)}\right]^{-1} R_{h_l}^{ij} = \phi^{j}\,. \tag{2.17}$$

The physical states invariant under the orbifold action and which can be identified as gauge bosons are the following:

$$A_\mu = \begin{pmatrix} A_\mu^1 & 0 & 0 & 0 \\ 0 & A_\mu^2 & 0 & 0 \\ 0 & 0 & A_\mu^3 & 0 \\ 0 & 0 & 0 & A_\mu^4 \end{pmatrix} \tag{2.18}$$

showing that the gauge group is now $\mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \mathrm{U}(N_3) \times \mathrm{U}(N_4)$, while the scalars of the twelve chiral multiplets can be organized in the following matrices:

$$\phi^1 = \begin{pmatrix} 0 & a_{i_1,\bar{j}_2} & 0 & 0 \\ a_{i_2,\bar{j}_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{i_3,\bar{j}_4} \\ 0 & 0 & a_{i_4,\bar{j}_3} & 0 \end{pmatrix}, \qquad \phi^2 = \begin{pmatrix} 0 & 0 & a_{i_1,\bar{j}_3} & 0 \\ 0 & 0 & 0 & a_{i_2,\bar{j}_4} \\ a_{i_1,\bar{j}_3} & 0 & 0 & 0 \\ 0 & a_{i_4,\bar{j}_2} & 0 & 0 \end{pmatrix}$$
$$\phi^3 = \begin{pmatrix} 0 & 0 & 0 & a_{i_1,\bar{j}_4} \\ 0 & 0 & a_{i_2,\bar{j}_3} & 0 \\ 0 & a_{i_3,\bar{j}_2} & 0 & 0 \\ a_{i_1,\bar{j}_4} & 0 & 0 & 0 \end{pmatrix} \tag{2.19}$$

where $i_I = 1, \ldots, N_I$ and the bar on the index j_I picks in the antifundamental representation of the group. Since we preserve $\mathcal{N}=1$ it is sufficient to consider the orbifold action on the NS sector and to deduce via supersymmetry the fermionic spectrum in the R sector.

In table 1 we summarize the spectrum of the chiral superfields according to their transformation properties under each gauge group.

It can be written compactly:

$$\Phi^1 \equiv (N_1, \overline{N}_2) + (N_2, \overline{N}_1) + (N_3, \overline{N}_4) + (N_4, \overline{N}_3)\,, \tag{2.20}$$

$$\Phi^2 \equiv (N_1, \overline{N}_3) + (N_2, \overline{N}_4) + (N_3, \overline{N}_1) + (N_4, \overline{N}_2)\,, \tag{2.21}$$

$$\Phi^3 \equiv (N_1, \overline{N}_4) + (N_2, \overline{N}_3) + (N_3, \overline{N}_2) + (N_4, \overline{N}_1)\,. \tag{2.22}$$

As expected, we have a vector-like theory which does not suffer of any gauge anomaly and it is well-defined. The vectors together with the relative gauginos (R-sector) are all living

U(N_1)	U(N_2)	U(N_3)	U(N_4)
$\square$	$\overline{\square}$	1	1
$\overline{\square}$	$\square$	1	1
$\square$	1	$\overline{\square}$	1
$\overline{\square}$	1	$\square$	1
$\square$	1	1	$\overline{\square}$
$\overline{\square}$	1	1	$\square$
1	$\square$	$\overline{\square}$	1
1	$\overline{\square}$	$\square$	1
1	$\square$	1	$\overline{\square}$
1	$\overline{\square}$	1	$\square$
1	1	$\square$	$\overline{\square}$
1	1	$\overline{\square}$	$\square$

Table 1: Spectrum of the chiral superfields. $\square$ and $\overline{\square}$ denotes respectively the fundamental and the antifundamental of each gauge group.

in the adjoint representation of each group. The corresponding one-loop β-functions are:

$$\beta(g_1) = -\frac{g_1^3}{16\pi^2}(3N_1 - N_2 - N_3 - N_4)\,, \tag{2.23}$$

$$\beta(g_2) = -\frac{g_2^3}{16\pi^2}(3N_2 - N_1 - N_3 - N_4)\,, \tag{2.24}$$

$$\beta(g_3) = -\frac{g_3^3}{16\pi^2}(3N_3 - N_1 - N_2 - N_4)\,, \tag{2.25}$$

$$\beta(g_4) = -\frac{g_4^3}{16\pi^2}(3N_4 - N_1 - N_2 - N_3)\,. \tag{2.26}$$

It is worth noticing that all of them vanish when $N_1 = N_2 = N_3 = N_4$.

2.3 D3-D7 open string spectrum on $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$: the matter

In this subsection we introduce D7-branes in order to add extra chiral matter to the gauge theory living on the stack of the D3-branes discussed previously. The matter is given by the open strings with one end attached to the D3-branes and the other one to the D7-brane. Physical states associated to these strings transform under the fundamental representation of the gauge group, thus providing chiral matter fields while the number of D7-branes can be regarded as a flavor index for them. In particular, we will consider configurations in which the D7-branes extend in the directions $x^0, \ldots, x^3, x^6, \ldots, x^9$, i.e. partially along the orbifold, while the D3-branes remain in the directions $x^0, \ldots x^3$, being localized completely in the D7-brane world-volume. The configuration is represented in table 2. Also in this case we are interested in studying the bosonic spectrum in the NS sector and the fermionic one in the R sector. Supersymmetry allows one to deduce one sector from the other. Let us therefore consider only the NS one. In this case the zero-point energy is zero. Furthermore the open strings can have four different boundary conditions along the directions x^α with

	X0	X1	X2	X3	X4	X5	X6	X7	X8	X9
D3	−	−	−	−	•	•	•	•	•	•
D7	−	−	−	−	•	•	−	−	−	−

Table 2: Configuration of the D3-D7 branes system.

$\alpha = 6, 7, 8, 9$ and therefore they have zero modes along these directions that generate $2^{4/2} = 4$ degenerate ground states, which we label by their "spins" in the z^2 and z^3 planes:

$$\lambda_{37}|s_2, s_3\rangle \,, \tag{2.27}$$

with λ_{37} being a Chan-Paton factor and $s_2 = s_3 = \pm 1/2$, as implied by the GSO projection. It turns out that a consistent definition of the orbifold action on the physical states is [18]:

$$\left[\gamma^{(D3)}_{h_i}\right] \lambda_{37} \left[\gamma^{(D7)}_{h_i}\right]^{-1} S_{h_i}(s_2, s_3)|s_2, s_3\rangle = \lambda'_{37}|s_2, s_3\rangle \tag{2.28}$$

with $S_{h_i}(s_2, s_3) = e^{i\pi(s_2 b^2_{h_i} + s_3 b^3_{h_i})}$ and

$$\begin{aligned}
\gamma^{(D7)}_e &= \mathrm{diag}(\mathbf{1}_{M_1,M_1}, \mathbf{1}_{M_2,M_2}, \mathbf{1}_{M_3,M_3}, \mathbf{1}_{M_4,M_4}) \,,\\
\gamma^{(D7)}_{h_1} &= \mathrm{diag}(\mathbf{1}_{M_1,M_1}, \mathbf{1}_{M_2,M_2}, -\mathbf{1}_{M_3,M_3}, -\mathbf{1}_{M_4,M_4}) \,,\\
\gamma^{(D7)}_{h_2} &= e^{-i\frac{\pi}{2}}\, \mathrm{diag}(\mathbf{1}_{M_1,M_1}, -\mathbf{1}_{M_2,M_2}, \mathbf{1}_{M_3,M_3}, -\mathbf{1}_{M_4,M_4}) \,,\\
\gamma^{(D7)}_{h_3} &= -e^{-i\frac{\pi}{2}}\, \mathrm{diag}(\mathbf{1}_{M_1,M_1}, -\mathbf{1}_{M_2,M_2}, -\mathbf{1}_{M_3,M_3}, \mathbf{1}_{M_4,M_4}) \,,
\end{aligned} \tag{2.29}$$

where M_I represents the number of fractional D7 branes of type I. The signs and the phase factors introduced in (2.29) are consistent with group properties of $\mathbb{Z}_2 \times \mathbb{Z}_2$ i.e. $h_1 \cdot h_2 = h_3$ and $h_i^2 = 1$.

In the same way we can study the open string spectrum of the D7-D3 open strings. In this case the states in the NS sector left invariant by the orbifold projection are the ones satisfying:

$$\left[\gamma^{(D7)}_{h_i}\right] \lambda_{73} \left[\gamma^{(D3)}_{h_i}\right]^{-1} S_{h_i}(s_2, s_3) = \lambda_{73} \,. \tag{2.30}$$

The physical states are those invariant under the orbifold action. The physical spectrum (containing both bosons and fermions coming from the R sector) is:

$$\begin{aligned}
\chi \equiv \Phi_{37} + \Phi_{73} \sim{}& (N_1, \overline{M}_3) + (N_2, \overline{M}_4) + (N_3, \overline{M}_1) + (N_4, \overline{M}_2) + \\
&+ (M_4, \overline{N}_1) + (M_3, \overline{N}_2) + (M_2, \overline{N}_3) + (M_1, \overline{N}_4) \,.
\end{aligned} \tag{2.31}$$

The whole spectrum including both D3-D7 and D7-D3 open strings contains eight chiral superfields as shown in table 3 and, in general, is not gauge anomaly free. However a simple consistent vector-like theory is obtained for $M_1 = M_2$ and $M_3 = M_4$. This spectrum yields the following contributions to the β-functions of the various gauge groups:

$$\Delta\beta(g_1) = \Delta\beta(g_2) = \frac{g^3}{16\pi^2}\left[\frac{1}{2}(M_3 + M_4)\right] \,, \tag{2.32}$$

$$\Delta\beta(g_3) = \Delta\beta(g_4) = \frac{g^3}{16\pi^2}\left[\frac{1}{2}(M_1 + M_2)\right] \,. \tag{2.33}$$

D3 gauge-groups				D7 global-symmetries			
U(N_1)	U(N_2)	U(N_3)	U(N_4)	U(M_1)	U(M_2)	U(M_3)	U(M_4)
$\square$	1	1	1	1	1	$\overline{\square}$	1
$\overline{\square}$	1	1	1	1	1	1	$\square$
1	$\square$	1	1	1	1	1	$\overline{\square}$
1	$\overline{\square}$	1	1	1	1	$\square$	1
1	1	$\square$	1	$\overline{\square}$	1	1	1
1	1	$\overline{\square}$	1	1	$\square$	1	1
1	1	1	$\square$	1	$\overline{\square}$	1	1
1	1	1	$\overline{\square}$	$\square$	1	1	1

Table 3: Spectrum for the D3-D7 and D7-D3 systems.

Finally in $\mathcal{N}=1$ supersymmetry the interaction between the different chiral fields can be encoded in the following cubic superpotential [1] schematically written as:

$$W = \mathrm{Tr}\left(\Phi_1[\Phi_2,\Phi_3]\right) + \sum_{i=1}^{3} \mathrm{Tr}\left(\Phi^i\,\chi^2\right). \tag{2.34}$$

It is important to observe that if we turn off three of the gauge groups by taking $N_2 = N_3 = N_4 = 0$ and $M_1 = M_2 = 0$ while keeping $N_1 = N$ and $M_3 = M_4 = M$ non zero the resulting gauge theory is exactly $\mathcal{N}=1$ super QCD with matter.

3. The fractional D3/D7 bound state

3.1 The boundary state approach

In this subsection we analyze a D3/D7 fractional branes system located at the fixed point of the orbifold $\mathbb{R}^{1,3} \times \mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ by using the boundary state approach.

The boundary state description for a fractional Dp-brane with a given number of directions parallel to the orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ is encoded in the following one-loop vacuum energy Z of the open strings stretched between two such Dp-branes:

$$Z = \int_0^\infty \frac{ds}{s}\,\mathrm{Tr}_{\mathrm{NS-R}}\left[P_{\mathrm{GSO}}\left(\frac{\mathbf{1}+h_1+h_2+h_3}{4}\right)\mathrm{e}^{-2\pi s(L_0-a)}\right], \tag{3.1}$$

where $a = 0[1/2]$ in the R [NS] sector and the orbifold group elements $(\mathbf{1},\cdots,h_3)$ act both on the oscillators and on the Chan-Paton factors of the open strings.

Alternatively, the interaction between two fractional Dp-branes can be more easily studied by performing in eq. (3.1) the modular transformation $s \to 1/s$, that leads to the closed string channel, and by rewriting Z as a matrix element between two boundary states $|Bp\rangle$:

$$Z = \langle Bp|\mathcal{D}|Bp\rangle\,, \tag{3.2}$$

JHEP09(2002)010

with $\mathcal{D}$ being the closed string propagator. Our boundary states are the sum of four terms, one untwisted obtained by performing the modular transformation on the first term in eq. (3.1), i.e. the one associated to the identity of the discrete group, and three twisted sectors associated to the three non trivial group elements h_i.

The sign in front of each twisted sector is determined by eqs. (2.11) and therefore we have four boundary states describing the four different fractional branes that we can have in this orbifold. They are given by:

$$\begin{aligned}
|Bp\rangle_1 &= |Bp\rangle_u + |Bp\rangle_{t_1} + |Bp\rangle_{t_2} + |Bp\rangle_{t_3}\,, \\
|Bp\rangle_2 &= |Bp\rangle_u + |Bp\rangle_{t_1} - |Bp\rangle_{t_2} - |Bp\rangle_{t_3}\,, \\
|Bp\rangle_3 &= |Bp\rangle_u - |Bp\rangle_{t_1} + |Bp\rangle_{t_2} - |Bp\rangle_{t_3}\,, \\
|Bp\rangle_4 &= |Bp\rangle_u - |Bp\rangle_{t_1} - |Bp\rangle_{t_2} + |Bp\rangle_{t_3}\,,
\end{aligned} \tag{3.3}$$

where p is the integer relative to the Dp-brane and u labels the untwisted sector while t_i with $i = 1, \ldots, 3$ label the twisted states.

The untwisted part of the boundary state, as already noticed in ref. [19], has the same structure as in flat space but with an extra $1/2$ factor due to the $1/4$ factor of the orbifold projector appearing in eq. (3.1). The other terms in eqs. (3.3), instead, apart an extra factor $1/\sqrt{2}$ due again to the orbifold projector, are the same as those for the orbifold $\mathbb{C}^2/\mathbb{Z}_2$.

Let us consider for instance the part in eq. (3.1) containing just the h_1 group element. Since h_1 is the generator of $\mathbb{Z}_2$ acting as a reflection along the directions z_2 and z_3, the associated twisted boundary states are the ones corresponding to the twisted sector of the $\mathbb{Z}_2$ orbifold acting on these directions. Similar arguments can be repeated for the other two elements h_2 and h_3 acting respectively on $(z_1,\, z_3)$ and on $(z_1,\, z_2)$.

Finally for the orbifold $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ we can directly use the twisted boundary state given in ref. [16] for the orbifold $\mathbb{C}^2/\mathbb{Z}_2$ and multiply it for an additional factor $1/\sqrt{2}$.

Using the same logic we derive the couplings between the boundary states and the closed string states. These couplings are $1/\sqrt{2}$ of the corresponding ones for the orbifolds $\mathbb{C}^2/\mathbb{Z}_2$ while we now have three twisted sectors. Following ref. [19] we deduce the couplings of the fractional D3-branes to the fluctuation of the h_{MN}, of the untwisted four-form potential C_4 and of the twisted fields $\widetilde{b}^i$ and A^i_{0123}:[3]

$$\begin{aligned}
{}_u\langle B3|h\rangle &= -\frac{T_3}{2\kappa_{\rm orb}}\, V_4\, h_\mu{}^\mu\,, & {}_u\langle B3|C_4\rangle &= \frac{T_3}{2\,\kappa_{\rm orb}}\, V_4\, C_{0123} \\
{}_{t_i}\langle B3|\widetilde{b}^i\rangle &= -\frac{T_3}{2\,\kappa_{\rm orb}}\, \frac{V_4}{2\pi^2\alpha'}\, \widetilde{b}^i\,, & {}_{t_i}\langle B3|A^i_4\rangle &= \frac{T_3}{2\,\kappa_{\rm orb}}\, \frac{V_4}{2\pi^2\alpha'}\, A^i_{0123}
\end{aligned} \tag{3.4}$$

where for a general p $T_p = \sqrt{\pi}/(2\pi\sqrt{\alpha'})^{p-3}$, $\kappa_{\rm orb} = 2\,\kappa = 16\pi^{7/2}(\alpha')^2 g_s$, V_4 is the (infinite) world-volume of the D3-brane and we defined $b^i = b_0 + \widetilde{b}^i$, being b_0 the background value of the B_2-flux [24]:

$$\int_{\mathcal{C}_i} B_2 = 4\pi^2\alpha' \frac{1}{2} \equiv b_0$$

[3] The couplings written in the paper [19] are recovered when using the relation between κ and $\kappa_{\rm orb}$.

for any i. From the above couplings one can write the boundary action for a fractional D3-brane of type I on this orbifold:

$$S_{b,I}^{\mathrm{D3}} = -\frac{T_3}{2\kappa_{\mathrm{orb}}}\int d^4x\sqrt{-\det G_{\mu\nu}}\left[1+\frac{1}{2\pi^2\alpha'}\sum_{i=1}^{3}\mathcal{F}_I^i\,\widetilde{b}^i\right]+$$
$$+\frac{T_3}{2\kappa_{\mathrm{orb}}}\int\left[C_4\left(1+\frac{1}{2\pi^2\alpha'}\sum_{i=1}^{3}\mathcal{F}_I^i\,\widetilde{b}^i\right)+\frac{1}{2\pi^2\alpha'}\sum_{i=1}^{3}\mathcal{F}_I^i\,A_4^i\right]. \tag{3.5}$$

The quantities $\mathcal{F}_I^i$ take in account the different signs appearing in the definition of the boundary states (3.3) and their explicit form is:

$$\vec{\mathcal{F}}_1=(+1,+1,+1)\qquad \vec{\mathcal{F}}_2=(+1,-1,-1)\qquad \vec{\mathcal{F}}_3=(-1,+1,-1)\qquad \vec{\mathcal{F}}_4=(-1,-1,+1)\,. \tag{3.6}$$

In the case of a D7-brane, which is transverse only to the direction z_1 and therefore partially extended along the orbifold, we have to take into account the condition $M_1 = M_2$ and $M_3 = M_4$, that as previously discussed, leads to an anomaly-free gauge theory. The role played by this condition is to cancel, in the boundary state describing this compound system, the contributions coming from the twisted sectors t_2 and t_3 in eq. (3.3). Hence we consider only the couplings between the bulk fields and the twisted boundary state associated to the element h_1 of the orbifold group.

By computing the couplings between the D7-fractional branes and the bulk fields we get for the untwisted sector the following expressions:

$$\begin{aligned}
{}_u\langle B7|h\rangle &= -\frac{T_7}{2\kappa_{\mathrm{orb}}}V_8 h_\alpha^{\ \mu}\,, \qquad {}_u\langle B7|C_8\rangle = \frac{T_7}{2\,\kappa_{\mathrm{orb}}}V_8\,C_{0,1,2,3,6,7,8,9}\\
{}_u\langle B7|\phi\rangle &= \frac{T_7}{2\,\kappa_{\mathrm{orb}}}V_8\phi\,,
\end{aligned} \tag{3.7}$$

where ϕ is the dilaton and C_8 is the Ramond-Ramond 8-form potential, while for the twisted fields we get:

$${}_{t_1}\langle B7|\widetilde{b}^1\rangle = -\frac{T_3}{2\,\kappa_{\mathrm{orb}}}\frac{V_4}{8\pi^2\alpha'}\,\widetilde{b}^1\,,\qquad {}_{t_1}\langle B7|A_4^1\rangle = \frac{T_3}{2\,\kappa_{\mathrm{orb}}}\frac{V_4}{8\pi^2\alpha'}\,A_{0123}^1\,. \tag{3.8}$$

Using the couplings (3.7) and (3.8) one can infer the form of the linearized world-volume action for a D7 of type I:

$$S_{b,I}^{\mathrm{D7}} = -\frac{T_7}{2\,\kappa_{\mathrm{orb}}}\int d^8x e^{\phi}\sqrt{-\det G_{\rho\sigma}}+\frac{T_7}{2\,\kappa_{\mathrm{orb}}}\int C_8-$$
$$-\frac{T_3}{2\,\kappa_{\mathrm{orb}}}\frac{1}{8\pi^2\alpha'}\int d^4x\sqrt{-\det G_{\rho\sigma}}\mathcal{F}_I^1\,\widetilde{b}^1+\frac{T_3}{2\,\kappa_{\mathrm{orb}}}\frac{1}{8\pi^2\alpha'}\int\mathcal{F}_I^1\,A_4^1+\cdots\,, \tag{3.9}$$

where ρ,σ run over $0,\ldots,3,6\ldots9$. Here we have enforced reparametrization invariance and the ellipses stay for terms of higher order in g_s which are not accounted for by the boundary state approach.

Equation (3.9) tells us that the boundary action for a D7-brane of type 1 is coincident with the one for a D7-brane of type 2; the same happens for type 3 and type 4. This means that, due to the anomaly free gauge condition, we have only two different kinds of fractional branes.

JHEP09(2002)010

We now study the bound state made of N_1, N_2, N_3 and N_4 D3-fractional branes respectively of type $1, \ldots, 4$ extended along the direction $x^0, \ldots, x^3$, and $M_1 = M_2$, $M_3 = M_4$ D7-fractional branes respectively of type $1, \ldots, 4$ (with $1, 2$ and $3, 4$ identified) parallel to the D3-fractional branes and extended along the directions $x^6, \ldots, x^9$ of the orbifold.

As explained in ref. [20, 21], the boundary state formalism allows to compute the asymptotic behavior for large distances of the various fields in the classical brane solution. In our case the boundary states of the D3/D7 system are just linear combinations of the ones describing the constituent branes. It follows that the asymptotic behavior of the classical solution is the sum of the behaviors generated by the single branes. We find that, at the leading order in g_s, the dilaton and the R-R potentials are:

$$\phi \simeq \frac{f_0(M) g_s}{2\pi} \log \rho_1 + \cdots , \qquad C_4 \simeq -\frac{Q_3}{r^4}\, dx^0 \wedge \cdots \wedge dx^3 + \cdots$$
$$C_8 \simeq \frac{f_0(M) g_s}{2\pi} \log \rho_1 \; dx^0 \wedge \cdots \wedge dx^3 \wedge dx^6 \cdots \wedge dx^9 + \cdots \tag{3.10}$$

where $\rho_i = \sqrt{z_i \bar{z}_i}\,/\epsilon$ for $i = 1, 2, 3$ and ϵ is a regulator. The metric is:

$$ds^2 \simeq H^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + H^{1/2} \left(e^{-\phi} \delta_{ab} dx^a dx^b + \delta_{\alpha\beta} dx^\alpha dx^\beta \right) + \cdots , \tag{3.11}$$

where

$$H = \left(1 + \frac{Q_3}{r^4} + \cdots \right), \qquad r = \sqrt{(x^4)^2 + \cdots + (x^9)^2}\,, \qquad Q_3 = 4\,\pi\,(\alpha')^2 g_s f_0(N)\,, \tag{3.12}$$

with $\mu, \nu = 0, \ldots, 3$, $a, b = 4, 5$ and $\alpha, \beta = 6, \ldots, 9$. Finally the asymptotic behavior of the twisted fields is given by (no sum over i):

$$\widetilde{b}^i \simeq 4\pi g_s \alpha' \left(f_i(N) + \frac{f_1(M)}{4} \delta_1^i \right) \log \rho_i\,, \qquad A^i \simeq 4\pi g_s \alpha' \left(f_i(N) + \frac{f_1(M)}{4} \delta_1^i \right) \log \rho_i\,. \tag{3.13}$$

In these formulas the factors $f_i(V)$ ($V = M, N$) take into account the different signs in front of the twisted boundary states (3.3) and their explicit form is:

$$f_0(V) = V_1 + V_2 + V_3 + V_4\,, \qquad f_1(V) = V_1 + V_2 - V_3 - V_4\,,$$
$$f_2(V) = V_1 - V_2 + V_3 - V_4\,, \qquad f_3(V) = V_1 - V_2 - V_3 + V_4\,. \tag{3.14}$$

In order to extend the previous analysis to all orders we should solve the complete equation of motion of type-IIB supergravity in this background, having as a source term the actions described by eqs. (3.5) and (3.9). This requires a suitable *ansatz* for the supergravity fields. The asymptotic ansatz, given by eqs. (3.10)–(3.13), is not the one that can provide the complete solution in the case of branes localized inside branes, which is exactly the configuration we are considering here [25].

In the following we will be interested in the classical profile of the RR-form of lower degree and therefore we rewrite the previous solution in terms of the axion C_0 and the twisted scalar c related respectively to the Hodge duals of the RR-potentials C_8 and A_4^i. At first order in g_s we have:

$$^{*}dC_8 = -dC_0\,, \qquad ^{*6}dc^i = dA_4^i\,, \tag{3.15}$$

whose explicit solutions from eqs. (3.10) and (3.13) are:

$$C_0 \simeq \frac{f_0(N)}{2\pi} g_s \theta_1 \,, \qquad c^i \simeq -4\pi g_s \alpha' \left(f_i(N) + \frac{f_1(M)}{4} \delta_1^i \right) \theta_i \,, \tag{3.16}$$

where $\theta_i = \tan^{-1} x^{2i+3}/x^{2i+2}$ for $i = 1, 2, 3$. We combine together the dilaton, the axion and the twisted scalars b and c in two complex quantities relevant for the gravity dual:

$$\tau \simeq C_0 + i e^{-\phi} = i - \frac{g_s}{2\pi} f_0(M) \log w_1 \,, \tag{3.17}$$

$$\gamma^{(1)} \simeq \tau b^1 + c^1 = i 2\pi^2 \alpha' + 4\pi g_s \alpha' i \left(f_1(N) + \frac{(f_1(M) - f_0(M))}{4} \right) \log w_1 \,, \tag{3.18}$$

$$\gamma^{(2)} \simeq \tau b^2 + c^2 = i 2\pi^2 \alpha' + 4\pi g_s \alpha' i \left(f_2(N) \log w_2 - \frac{f_0(M)}{4} \log w_1 \right) , \tag{3.19}$$

$$\gamma^{(3)} \simeq \tau b^3 + c^3 = i 2\pi^2 \alpha' + 4\pi g_s \alpha' i \left(f_3(N) \log w_3 - \frac{f_0(M)}{4} \log w_1 \right) , \tag{3.20}$$

where we have defined $w_i = z_i/\epsilon$.

We see from eqs. (3.17)–(3.20) that the dilaton and the γ's are holomorphic functions of the w_i's, at least at the first order in g_s. This is a property common to all classical solutions having fractional branes as a source and it is a consequence of supersymmetry [26].

3.2 Stability of the system and massless tadpole cancellation

We now investigate the stability of the system related to the one-loop massless tadpole cancellation. The generic boundary state describing our D3/D7 fractional system is the following linear combination of the boundary states introduced in the previous paragraph:

$$|B3/7\rangle_{I,J} = |B3\rangle_I + |B7\rangle_J \,. \tag{3.21}$$

Two D-branes interact at the tree level via the exchange of closed strings and the interaction is:

$$\frac{\alpha'\pi}{2} \int_0^\infty dt \; {}_{I,J}\langle B3/7| e^{-\pi t \left(L_0 + \bar{L}_0 - 2a \right)} |B3/7\rangle_{I,J} \,, \tag{3.22}$$

where $a = 0$ in the twisted sectors. Due to the BPS no-force condition even before explicitly evaluating the previous expression we expect a vanishing result. This is consistent with our bound state which preserves $\mathcal{N} = 1$ and with the massless tadpole cancellation requirement.

By separating the various contributions in the correlator (3.22) and considering only the twisted sectors encoding the orbifold information we obtain:

$$\begin{aligned} &\frac{\alpha'\pi}{2} \int_0^\infty dt \; {}_{t_i}\langle Bp| e^{-\pi t \left(L_0 + \bar{L}_0 - 2a \right)} |Bp\rangle_{t_i} = \\ &= \frac{\alpha'\pi}{4} \frac{T_{r_i}^2 V_{r_i+1}}{2^{s_i} (2\pi^2\alpha')^{\frac{9-r_i}{2}}} \int_0^\infty \frac{dt}{t^{\frac{5-r_i}{2}}} \prod_{n=1}^{\infty} \frac{\left(1 + e^{-2n\pi t}\right)^4 \left(1 + e^{-(2n-1)\pi t}\right)^4}{\left(1 - e^{-2n\pi t}\right)^4 \left(1 - e^{-(2n-1)\pi t}\right)^4} \left(1|_{\mathrm{NS-NS}} - 1|_{\mathrm{RR}} \right) , \end{aligned} \tag{3.23}$$

$$\begin{aligned} &\frac{\alpha'\pi}{2} \int_0^\infty \frac{dt}{t} \; {}_{t_i}\langle B3| e^{-\pi t \left(L_0 + \bar{L}_0 - 2a \right)} |B7\rangle_{t_i} = \\ &= \frac{\alpha'\pi}{16} \delta_{t_i, t_1} \frac{T_3^2 V_4}{(2\pi^2\alpha')^3} \int_0^\infty \frac{dt}{t} \prod_{n=1}^{\infty} \frac{\left(1 + e^{-2n\pi t}\right)^4 \left(1 - e^{-(2n-1)\pi t}\right)^4}{\left(1 - e^{-2n\pi t}\right)^4 \left(1 + e^{-(2n-1)\pi t}\right)^4} \left(1|_{\mathrm{NS-NS}} - 1|_{\mathrm{RR}} \right) , \end{aligned} \tag{3.24}$$

where r_i's are the world-volume directions outside the sector i of the orbifold and s_i's are the world-volume directions inside the sector i of the orbifold. By sector i of the orbifold we mean the two complex coordinates non trivially transformed by the group element h_i and hence we have for the D3-brane $r_i = 3$ ($s_i = 0$) for $i = 1, 2, 3$ while for the D7-brane $r_1 = 3$, $r_2 = r_3 = 5$ ($s_1 = 4$, $s_2 = s_3 = 2$). Equations (3.23) and (3.24) show the cancellation between the contributions due respectively to the NS-NS and the RR sectors. Each of the two term corresponds to a one-loop massless tadpole. For such a term, we see that in the limit $t \to \infty$ we have only logarithmic divergences for the contributions coming from sandwiching the closed string propagator with the D3 boundary state since $r_i = 3$. The same holds true when considering one boundary state of type D3 and the other of type D7. In the spirit of the open/closed sting duality these logarithmic divergences are related to the UV divergences of the gauge theory. When sandwiching the propagator between two D7 boundary states we observe in the twisted sectors t_2 and t_3 the rise of linear divergences which are dangerous since they lead to supersymmetry and/or Poincaré breaking [27].

As can be seen from the states in eqs. (3.3) this divergence can be cured by imposing $M_1 = M_2$ and $M_3 = M_4$. Such a condition yields, as expected, a consistent gauge theory free from gauge anomalies as seen in section 2.

4. Supergravity analysis of the dual gauge theory anomalies

In this section we discuss in detail from the dual supergravity point of view the trace and chiral anomalies of the gauge theory supported by a bound state made of D3/D7 branes.

As mentioned above, in our case the gauge theory living on the D3-world volume is an $\mathcal{N} = 1$ super Yang-Mills theory with gauge group $\mathrm{SU}(N_1) \times \mathrm{SU}(N_2) \times \mathrm{SU}(N_3) \times \mathrm{SU}(N_4)$. Here we do not consider the effects of the abelian U(1) factors.[4] The theory has twelve chiral multiplets in the bifundamental representation of the gauge group due to the D3/D3 open strings and chiral matter multiplets, both in the fundamental and in the antifundamental representation, due to the D3/D7 strings.

At the classical level this theory is invariant under an abelian $\hat{\mathrm{U}}(1)_R$ and the scale symmetry. We note that it is always possible to define a new anomalous free $\mathrm{U}(1)_R$ symmetry by considering a suitable choice of the chiral super-multiplets. The scalars of the gauge theory have canonical dimension one and R-$\hat{\mathrm{U}}(1)_R$ charge 2/3 as it follows from the cubic superpotential, therefore under a combined chiral and scale transformation with parameters respectively α and σ they transform as:

$$\phi' = e^{\sigma + i\frac{2}{3}\alpha}\phi\,. \tag{4.1}$$

These global symmetries are broken at the quantum level by anomalies, and the respective currents are given by:

$$\partial_\mu D^\mu = \frac{\beta(g_{\mathrm{YM}})}{2\, g_{\mathrm{YM}}^3} F^a_{\mu\nu} F^{a\,\mu\nu}\,, \tag{4.2}$$

$$\partial_\mu J^\mu_R = \sum_{\text{fermions}} \frac{Q_R\, C_2(T)}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu}\,, \tag{4.3}$$

[4] The effects are subleading in the large-N limit.

where Q_R is the R-charge of the fermions, $C_2(T)$ is the quadratic Casimir in the representation T of the gauge group and $\tilde{F}^{a\,\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{a\,\rho\sigma}$. For infinitesimal transformations these equations imply [28]:

$$\frac{1}{g^2_{\rm YM}} \to \frac{1}{g^2_{\rm YM}} - \frac{2\beta(g)}{g^3_{\rm YM}}\sigma \qquad \theta_{\rm YM} \to \theta_{\rm YM} - 2\alpha \sum_{\rm fermions} Q_R C_2(T)\,. \tag{4.4}$$

In a supersymmetric theory, such as the one we are considering, these two anomalies are in the same supermultiplet; it follows that, in a renormalization scheme preserving the ABJ theorem, the β-function appearing in the previous expression is the one at the 1-loop level. Indeed in the present case, by using the spectrum discussed in section 2, one obtains the 1-loop β-function for each $\mathrm{U}(N_I)$ leading to:

$$\partial_\mu D^\mu_I = -\frac{1}{2(4\pi)^2}\left[3\,N_I - \sum_{J\neq I=1}^{4} N_J - \frac{1}{2}\left(M_{I+2}+M_{-I+5}\right)\right] F^a_{(I)\,\mu\nu}F^{a\,\mu\nu}_{(I)}\,, \tag{4.5}$$

where M_I satisfies the condition $M_{I+4} \equiv M_I$.

Analogously, since the R-charge for the gauginos is one while for the chiral fermions it is $-1/3$, the chiral anomaly for each gauge group turns out to be:

$$\partial_\mu J^\mu_{R,(I)} = \frac{1}{(4\pi)^2}\left[N_I - \frac{1}{3}\sum_{J\neq I=1}^{4} N_J - \frac{1}{6}\left(M_{I+2}+M_{-I+5}\right)\right] F^a_{(I)\,\mu\nu}\tilde{F}^{a\,\mu\nu}_{(I)}\,. \tag{4.6}$$

By defining the well-known complex gauge coupling

$$\tau_{\rm YM} = \frac{4\,\pi}{g^2_{\rm YM}}i + \frac{\theta_{\rm YM}}{2\pi}\,, \tag{4.7}$$

and by using the expression of the one-loop beta functions and chiral anomaly together with eq. (4.4), one can easily see that $\tau_{\rm YM}$ transforms as:

$$\tau^I_{\rm YM} \to \tau^I_{\rm YM} + \frac{i}{2\pi}\left[3\,N_I - \sum_{J\neq I=1}^{4} N_J - \frac{1}{2}\left(M_{I+2}+M_{-I+5}\right)\right]\left(\sigma + \frac{2}{3}i\alpha\right)\,. \tag{4.8}$$

Let us now analyze how the scale and chiral anomalies are realized in supergravity and to this aim let us consider the Dirac-Born-Infeld action and the Wess-Zumino term for a stack of N_I fractional D3-branes given by eq. (3.5). Turning on a gauge field on the world-volume of the branes and expanding the boundary action in the supergravity background up to quadratic terms in the derivatives one gets:

$$\begin{aligned} S^I_{\rm gauge} = &-\frac{1}{16\pi g_s}\int d^4x\sqrt{-\det G}\, e^{-\phi} G^{\mu\rho}G^{\nu\sigma}\,\frac{1}{4}F^a_{\mu\nu}F^a_{\rho\sigma}\left[1 + \frac{1}{2\pi^2\alpha'}\sum_{i=1}^{3}\mathcal{F}^i_I\,\tilde{b}^i\right] + \\ &+ \frac{1}{64\pi g_s}\int d^4x\left[C_0\left(1+\frac{1}{2\pi^2\alpha'}\sum_{i=1}^{3}\mathcal{F}^i_I\,\tilde{b}^i\right) + \frac{1}{2\pi^2\alpha'}\sum_{i=1}^{3}\mathcal{F}^i_I c^i\right]F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\,, \end{aligned} \tag{4.9}$$

where for simplicity we dropped the index I of the gauge fields and the metric G is the pull-back to the brane world-volume. The previous boundary action, when evaluated in the supergravity background obtained in the previous section, yields the (gauge part of) $\mathcal{N}=1$ super YM theory with gauge group $\mathrm{SU}(N_I)$. More explicitly one obtains:

$$S^I_{\rm gauge} = -\frac{1}{g^2_{\rm YM}}\int d^4x \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{\theta_{\rm YM}}{32\pi^2}\int d^4x F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu}\,, \tag{4.10}$$

where the indices are raised by the flat metric $\eta_{\mu\nu}$ and

$$\frac{1}{g^2_{\rm YM}} = \frac{1}{16\pi g_s}e^{-\phi}\left[1+\frac{1}{2\pi^2\alpha'}\sum_{i=1}^{3}\mathcal{F}^i_I\,\tilde{b}^i\right], \tag{4.11}$$

$$\theta_{\rm YM} = \frac{\pi}{2\,g_s}\left[C_0\left(1+\frac{1}{2\pi^2\alpha'}\sum_{i=1}^{3}\mathcal{F}^i_I\,\tilde{b}^i\right)+\frac{1}{2\pi^2\alpha'}\sum_{i=1}^{3}\mathcal{F}^i_I\,c^i\right]. \tag{4.12}$$

It is worthwhile to observe that in eq. (4.10) all the dependence on the function H of the metric disappears. We have also to include matter in the fundamental and bifundamental representations of the gauge groups. Then, by using eqs. (3.10) and (3.13) the gauge coupling and the theta angle turn out to be related to the bulk fields by the following expressions:

$$\frac{1}{g^2_{\rm YM}} = \frac{1}{16\pi g_s}+\frac{1}{8\,\pi^2}\sum_{i=1}^{3}\left[\mathcal{F}^i_I\,f_i(N)+\frac{\mathcal{F}^1_I\,f_1(M)-f_0(M)}{4}\delta^i_1\right]\log\rho_i\,, \tag{4.13}$$

$$\theta_{\rm YM} = -\sum_{i=1}^{3}\left[\mathcal{F}^i_I\,f_i(N)+\frac{\mathcal{F}^1_I f_1(M)-f_0(M)}{4}\delta^i_1\right]\theta_i\,. \tag{4.14}$$

We note that the method considered above does not rely on the probe technique. The ingredients we are using are the holographic identification among the world-volume fields and the bulk supergravity quantities. In this spirit one has to consider the relations (4.13) and (4.14) as directly generated by the low-energy limit of the world-volume action of the fractional branes.

Analogously with the gauge theory we combine the previous equations and construct the supergravity realization of the complex coupling $\tau_{\rm YM}$:

$$\tau^I_{\rm YM} = \frac{i}{4\,g_s}+\frac{i}{2\,\pi}\sum_{i=1}^{3}\mathcal{F}^i_I f_i(N)\log w_i+\frac{i}{2\,\pi}\frac{\mathcal{F}^1_I\,f_1(M)-f_0(M)}{4}\log w_1\,. \tag{4.15}$$

In supergravity the coordinates z_i are holographically identified with the scalar components ϕ_i of the chiral superfield Φ_i. By using this identification we deduce that under a chiral and scale transformation the coordinates transverse to the brane transform according to the eq. (4.1), i.e. $z_i \to e^{\sigma+\frac{2}{3}i\alpha}z_i$. The classical profile of the twisted fields are not invariant under these transformations, showing the explicit breaking of the related symmetries. In particular the complex combination given in eq. (4.15) behaves as:

$$\tau^I_{\rm YM}\to\tau^I_{\rm YM}+\frac{i}{2\,\pi}\left[\sum_{i=1}^{3}\mathcal{F}^i_I f_i(N)+\frac{\mathcal{F}^1_I\,f_1(M)-f_0(M)}{4}\right]\left(\sigma+\frac{2}{3}i\alpha\right), \tag{4.16}$$

where

$$\begin{aligned}\mathcal{F}_1^1 f_1(M) - f_0(M) &= \mathcal{F}_2^1 f_1(M) - f_0(M) = -2(M_3 + M_4)\\ \mathcal{F}_3^1 f_1(M) - f_0(M) &= \mathcal{F}_4^1 f_1(M) - f_0(M) = -2(M_1 + M_2)\\ \sum_{i=1}^{3} \mathcal{F}_I^i f_i(N) &= 3N_I - \sum_{J\neq I=1}^{4} N_J\,. \end{aligned} \tag{4.17}$$

Combining eq. (4.16) with eq. (4.17) we recover the right β-function and chiral anomaly from the supergravity side.

In conclusion, by considering a suitable D3/D7 bound state living on an orbifold preserving $\mathcal{N}=1$, as $\mathbb{C}^3/(\mathbb{Z}_2\times\mathbb{Z}_2)$, we obtained a $\mathcal{N}=1$ supersymmetric theory with matter fields. We have studied the supergravity background determined by this brane configuration by using the boundary state formalism and from this classical solution we have got information about one-loop quantities of the gauge theory. In so doing, we have shown how the gauge/gravity correspondence may be used to analyze the quantum aspects, at least at perturbative levels, of the dual string-like standard model realized by considering branes at orbifold singularities in the spirit of the bottom-up approach. It would be nice to extend our analysis to the infrared regime of the gauge theory by finding a complete non-singular solution. This analysis would be performed by deforming, in an appropriate way, the orbifold singularities and finding a suitable ansatz valid also very near to the brane.

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